The Covariant Equations of Motions for the Fields and the Particles

Gyula I. Szász*

The covariant equations of motions for the fields and the particles is derived not within the in physics generally accepted energetic theory, but within the atomistic theory of matter www.atomsz.com. Such a basic theory was represented by Ludwig Boltzmann and is the opposite to the energetic basic concepts of Ernst Mach and Albert Einstein. Einstein was obliged to use different metrics of space-time for electrodynamics and for gravitation theory.

Within the Lagrange formalism the equations of motions are derived for interacting fields which propagate with c. The interactions are connected to point-like elementary particles with conserved elementary charges, q_i and g_i, in finite ranges of Minkowski space, Ω. The use of Ω is motivated since all measurements are carried out in finite ranges of space and time. Since the propagation of the fields is independent of the state of particles motions the interactions are non-conservative. This is a theory in which only the sources of the interaction fields are quantized by conserved elementary charges and leads to a quantum gravitodynamics similar to electrodynamics. Furthermore, I don't assume the knowledge of precise initial conditions since the knowledge of precise conditions of particles positions and velocities at any fixed times are connected to infinite exact measurements. I want describe four kinds of indivisible and indistinguishable elementary particles the electron (e), positron (p), proton (P) and the elton (E) which carry two kinds of conserved elementary charges. Therefore, the descriptions of particles have to be done with probability current densities, $j_i^{(n)\nu}(x)$, $\{x\} \varepsilon \Omega$, i = e,p,P,E, taking into account the different kinds of the elementary particles. The $j_i^{(n)\nu}(x)$ are four-vectors in Ω. The particle numbers conservations are expressed with the continuity equations

$$\partial_\nu j_i^{(n)\nu}(x) = 0, \quad i = e,p,P,E.$$

Since the elementary particles carry two kinds of the conserved electric and gravitational charges the charge conservation can also be expressed with continuity equations

$$\partial_\nu j^{(em)\nu}(x) = \Sigma_{i=e,p,P,E}\, q_i\, \partial_\nu j_i^{(n)\nu}(x) = 0,$$

$$\partial_\nu j^{(g)\nu}(x) = \Sigma_{i=e,p,P,E}\, g_i\, \partial_\nu j_i^{(n)\nu}(x) = 0.$$

I take into account that the elementary charges, $q_i = \{\pm e\}$, cause the electromagnetism and the elementary charges $g_i = \{\pm g{\cdot}m_e, \pm g{\cdot}m_P\}$ cause the gravitation. The invariant masses of electron and proton are given by m_e and m_P and $G = g^2/4\pi$ is the universal gravitational constant. The elementary charges play a double role: on the one side they characterize physically the elementary particles and on the other side they cause the interactions between the particles. Therefore, there is no need for the introduction of the weak- and strong-interactions in physics and since the elementary particles are indivisible, there is no need to consider quarks or other underlying particles which would compose e, p, P and E. Since I want to use only Lorentz invariant terms in the action-integral, the interaction terms between the continuous, non-conservative fields, $A^{(em)\nu}(x)$ and $A^{(g)\nu}(x)$ with $\nu = 0,1,2,3$, and the probability charge distributions, $j^{(em)\nu}(x)$ and $j^{(g)\nu}(x)$, must have the form

$+ j^{(em)}{}_{\nu}(x)\cdot A^{(em)\nu}(x)$ and $- j^{(g)}{}_{\nu}(x)\cdot A^{(g)\nu}(x)$, double occurring index, ν, mean summation.

Thus, the two fields, the electromagnetic and the gravitational fields, must be continuous four-vector-fields in Minkowski space. These fields are independent from each other then they are generated by different kinds of elementary charges. The different signs of the interaction terms are motivated since electric charges, q_i, with the same sign repulse each other and electric charges with different signs attract each other. For the gravitational charges, $g_{i,}$ it is converse. The gravitational charges with the same sign attract each other and gravitational charges with different signs repulse each other. The introduction of conserved elementary gravitational charges beside the conserved elementary electric charges leads to a completely new theoretical basic concept in physics. The consideration of two kinds of elementary gravitational charges is motivated by the use of the uniform static force equations between two bodies composed of elementary particles carrying two kinds of elementary charges

$\mathbf{F}^{(em)} = + \Sigma_l \Sigma_m{}^{l\neq m}\, q_l\cdot q_m\, (\mathbf{r}_l - \mathbf{r}_m)\, /4\pi|(\mathbf{r}_l - \mathbf{r}_m)|^3,$

$\mathbf{F}^{(g)} = - \Sigma_l \Sigma_m{}^{l\neq m}\, g_l\cdot g_m\, (\mathbf{r}_l - \mathbf{r}_m)\, /4\pi|(\mathbf{r}_l - \mathbf{r}_m)|^3.$

According to Newton's gravitation theory Euler has used a more simple equation in which only point-like bodies with gravitational charges of equal signs occurred and eventually bodies with different mass sizes,

$\mathbf{F}^{(Newton)} = - \Sigma_l \Sigma_m{}^{l\neq m}\, g^2\cdot M_l\cdot m_m\, (\mathbf{r}_l - \mathbf{r}_m)\, /4\pi|(\mathbf{r}_l - \mathbf{r}_m)|^3$, with M_l, $m_m > 0$.

The role of the universal gravitational constant, $G = g^2/4\pi$, is obviously. I would like point out that Euler has also used the knowledge of precise of the positions of point-like bodies carrying mass, but he didn't account with an eventually propagation of the gravitational field.

In my atomistic theory of matter based on four kinds of indivisible elementary particles I have extended the gravitational charges also to negative signs. Per convention, the proton and positron have positive gravitational charges, however with different magnitude

$g_P = + g\cdot m_P$, $g_p = + g\cdot m_e$ and $m_P/m_e = 1836.152$.

The elton and electron carry negative gravitational charges

$g_E = - g\cdot m_P$, $g_e = - g\cdot m_e$.

I notice, while only two kinds of elementary electric charges exists, the number of elementary gravitational charges is four corresponding to the four kinds of elementary particles e, p, P and E. Since the specific gravitational charge, $g > 0$, is the same for all four elementary particles and the universal gravitational constant is $G = g^2/4\pi$.

Now, I'm ready to construct an action-integral, I, in a Lorentz invariant fashion for the fields, for the particles and for the interactions. However, the action-integral is a probability density functional and it is not an expression for energy. I need yet the Faraday tensors of the fields

$$F^{(em)\mu\nu}(x) = \partial^{\mu}A^{(em)\nu}(x) - \partial^{\nu}A^{(em)\mu}(x),$$

$$F^{(g)\mu\nu}(x) = \partial^{\mu}A^{(g)\nu}(x) - \partial^{\nu}A^{(g)\mu}(x).$$

With the continuity equations of the elementary particles, i, multiplied by appropriate chosen constants, $m_i \cdot c$, the action-functional, I, has Lorentz invariant form

$$I = \int^{\Omega} (dx)^4 \{\Sigma_{i=e,p,P,E}\, m_i \cdot c\, \partial_{\nu}\, j_i^{(n)\nu}(x) - (F^{(em)}{}_{\mu\nu}(x)\, F^{(em)\mu\nu}(x) + F^{(g)}{}_{\mu\nu}(x)\, F^{(g)\mu\nu}(x))/4$$

$$- \Sigma_{i=e,p,P,E}\, q_i\, j_i{}^{(n)}{}_{\nu}(x)\, A^{(em)\nu}(x) + \Sigma_{i=e,p,P,E}\, g_i\, j_i{}^{(n)}{}_{\nu}(x)\, A^{(g)\nu}(x)\}.$$

Double occurring indices, ν, μ, are summations over ν, μ =0,1,2,3. The different terms correspond to elementary particles, to radiations of the fields and to interaction terms between elementary charges and fields. This Lagrange density is quite different from Einstein's stress-energy tensor which is thought to represent energy, rest masses, radiations and deformations of space and time. Einstein's metric for gravitation has also space-time singularities.

As mentioned, this action-integral/functional is not an expression for the energy. It depends only on five natural constants c, e, m_P, m_e and g. Since the integration is performed in finite ranges of Minkowski space, Ω, we need yet appropriate subsidiary conditions for the fields and for the particles within Ω and boundary conditions and on the closed surface of Ω. Generally, since the action should not depend on the boundary conditions, the boundary conditions can be considered as natural boundary conditions. The subsidiary conditions for the fields are given by the fact that the fields propagate within Ω with c

$$\partial_{\nu}\, A^{(em)\nu}(x) = 0,$$

$$\partial_{\nu}\, A^{(g)\nu}(x) = 0.$$

The condition for the electromagnetic field is known as Lorenz condition. Taking into account the subsidiary conditions, the equations of motions for the continuous vector-fields, $A^{(em)\nu}(x)$ and $A^{(g)\nu}(x)$, are derived in a straight forward way as Euler-Lagrange equations

$$\partial^{\mu}\, \partial_{\mu}\, A^{(em)\nu}(x) = + j^{(em)\nu}(x),$$

$$\partial^{\mu}\, \partial_{\mu}\, A^{(g)\nu}(x) = - j^{(g)\nu}(x).$$

The first equation is the well known Maxwell equation for the electromagnetic field. The equation of motion for the gravitation field is also a wave equation; however, the sign of the probability current density of the gravitational charges, $j^{(g)\nu}(x)$, is changed. Nevertheless, we have a unified description of the electromagnetic field and the gravitational field which is the result because I have used the conserved elementary gravitational charges in a similar way as the elementary electric charges. A notation should be making: Einstein has used the assumed equality of inertial and gravitational masses in order to determine his gravitational theory with a stress-energy tensor. As we'll see, in my theory the inertial and gravitational masses of bodies are different and furthermore, the point-like elementary particles cannot approach each other below 10^{-17} cm under the influence of their mutual interactions. With other words, I do also not use Einstein's mass-energy equivalence, $E = m\cdot c^2$, in my atomistic theory of matter. Opposite to Einstein, I don't use an energetic basic theory, but an atomistic basic theory.

For the particles, the connections between the probability current densities, $j_i^{(n)\nu}(x)$, and the particle number densities, $\rho_i(\mathbf{r},t)$, and the velocity current densities, $\mathbf{j}_i(\mathbf{r},t)$, are well known

$$j_i^{(n)\nu}(x) = (c\cdot\rho_i(\mathbf{r},t), \mathbf{j}_i(\mathbf{r},t)), \text{ i=e,p,P,E.}$$

The continuity equations are connected to the particle numbers conservation in a finite Ω

$$\int^{\Omega} (dx)^4\, \partial_\nu\, j_i^{(n)\nu}(x) = 0 \rightarrow \int_{t1}^{t2} dt\ d/dt\int^{V} dr^3\ \rho_i(\mathbf{r},t) = - \int_{t1}^{t2} dt \oint^{S} d\boldsymbol{s}\cdot\mathbf{j}_i(\mathbf{r},t) = N_i(t_2) - N_i(t_1).$$

During the time $t_2 - t_1$, the numbers of particles of kinds i in a finite volume V change only through the flow of particles, i, across the closed surface S of the volume V. Therefore, the particle number conservations correspond to an integral subsidiary conditions in V connected with boundary conditions on the closed surface S. Because the integral conditions in V, I denote this kinds of subsidiary conditions as **isopretic conditions** (equal integral valued condition). In the mathematics, it is known that at the variation of functionals with such kinds of subsidiary conditions induce Lagrange multipliers, λ_k: at the variation, terms must be added in a form that the subsidiary conditions multiplied by λ_k. In our case it is

$$\delta I + \delta\, \Sigma_k\, \lambda_k(\Sigma_i \int^{\Omega} (dx)^4\, \partial_\nu\, j_i^{(n)\nu}(x)) = 0.$$

Thus, the particle numbers conservation, N_i, cause Lagrange multipliers. Taking into account the Lagrange multipliers, the appearance of additional constants, λ_k, in the equation of particle motions is guaranteed. The researchers have never recognized the appearance of Lagrange multipliers in the equation of particle motions caused by subsidiary conditions in basic physics. In place of Lagrange multipliers, in the quantum theories the quantization of energy and the interactions with the Planck constant, h, are falsely declared (Planck, Einstein, Bohr).

In order to complete the derivation of the equation of particles motions, I have to go a step further; I have to express the probability density currents, $j_i^{(n)\nu}(x)$, with a bilinear form which can be as well normalized, as it is a Lorentz covariant vector too. The bilinear form guarantees the applicability of variation also for the "particle fields". For this reason, I use the well known Dirac spinors, $\psi_i(x)$ and their adjoint spinors, $\underline{\psi}_i(x) = \psi_i(x)^*\gamma^0$, with the known γ^ν matrixes

$$j_i^{(n)\nu}(x) = (c\cdot\rho_i(\mathbf{r},t), \mathbf{j}_i(\mathbf{r},t)) = c\ \underline{\psi}_i(x)\gamma^\nu\psi_i(x),\ \nu = 0,1,2,3 \text{ and i = e,p,P,E.}$$

I remark that the introduction of the Dirac spinors is neither used for the linearization of energy expressions, nor for spin 1/2. Putting these relations in the action-functional, I, for the variation and considering the spinors and the adjoint spinors for each particle kind, i, as independent variables, I got the following Euler-Lagrange equations for the particle motions

$(m_i \cdot c^2 - \lambda_j \partial_\nu \gamma^\nu) \psi_i(x) + q_i A^{(em)}{}_\nu(x) \gamma^\nu \psi_i(x) - g_i A^{(g)}{}_\nu(x) \gamma^\nu \psi_i(x) = 0$, i = e,p,P,E.

If all these equation are fulfilled and further, if the wave equations of both kinds of the fields with the propagation with c are also fulfilled, the variation is stationary. **All these equations are covariant equations**. For the particle motions, the equations are first order differential equations and are valid in each coordinate system in finite ranges of Minkowski space Ω. First order differential equations are appearing for the motions of particles because we didn't have used precise initial conditions for the particle positions and for the velocities at all. We remember, the continuous vector-functions, $A^{(em)\nu\,(x)}$ and $A^{(g)\nu}(x)$, represent non-conservative interactions and in the equations of particle motions the four components of $\psi_i(x)$

$\psi_i(x) = \{\psi_{i,0}(x), \psi_{i,1}(x), \psi_{i,2}(x), \psi_{i,3}(x)\}$

are mixed together. The spinors, $\psi_i(\mathbf{r},t)$, are normalizable at each time-points, t

$\int^V dr^3\, \underline{\psi}_i(\mathbf{r},t)\gamma^0\psi_i(\mathbf{r},t) = \int^V dr^3\, \Sigma_{l=0,1,2,3}\, \psi_{i,l}{}^*(\mathbf{r},t)\cdot\psi_{i,l}(\mathbf{r},t) = \int^V dr^3\, \rho_i(\mathbf{r},t) = N_i(t)$, i=e,p,P,E.

All equations are also valid in each coordinate systems in finite ranges of Minkowski space, Ω, and not only in the so called inertial systems as used in the theory of special relativity, or in accelerated coordinate systems of the theory of general relativity. Furthermore, the quantization conditions affect only the sources of the interaction fields and not the energy.

The temporary stationarity of particle states; the steady-states

We have another kind of stationary problem if we consider the temporary stationarity of particle states within Ω with conserved energies under the influence of the mutual interactions between particles and without radiations of energies in and out of Ω. For that, I choose up to now a special coordinate system in Ω in which the center of mass (COM) of $N = N_P + N_E + N_p + N_e$ elementary particles is at rest. At steady-states, not only the particle states must be temporary stationary but also the mutual fields as interaction fields between the particles. In this case, the relative coordinates between the particles must be considered and their relative velocities to each other, because the mutual interacting fields, $A'^{(em)}{}_\nu(x)$ and $A'^{(g)}{}_\nu(x)$, are depending on the relative coordinates and on the relative velocities. In center of mass at rest, all terms must be expressed with these relative quantities. In this sense, the probability current densities are also relative probability current densities, $j'_i{}^{(n)\nu}(x)$, with regard to COM at rest. The temporary stationarity of equations of motions in this special coordinate system must than fulfill the following equations for all particles reside in Ω with $i^2 = -1$

$(m_i \cdot c^2 - i \cdot \lambda_k' / 2\pi \cdot \partial_\nu \gamma^\nu)\psi'_i(x) + q_i A'^{(em)}{}_\nu(x) \gamma^\nu \psi'_i(x) - g_i A'^{(g)}{}_\nu(x) \gamma^\nu \psi'_i(x) = 0$, i = e,p,P,E.

In this case, I have design the Lagrange multipliers with a dash, λ_k' and divided by 2π. At steady-states all terms of the equations are stationary, that means all $\psi'_i(x)$'s have the form

$\psi'_i(x) = \exp(-i \cdot E \cdot t \cdot 2\pi/\lambda_k') \cdot \psi'_i(\mathbf{r})$ for all i = e,p,P,E,

with an overall time dependent factor $\exp(-i \cdot E \cdot t \cdot 2\pi/\lambda_k')$ and with one constant E. All particle probability current distributions,

$j'_i{}^{(n)\nu}(x) = (c \cdot \rho'_i(\mathbf{r},t), \mathbf{j'}_i(\mathbf{r},t)) = (c \cdot \rho'_i(\mathbf{r}), \mathbf{j'}_i(\mathbf{r}))$,

would then be independent of the time. We see, each λ_k' corresponds to a special constant value E, but this corresponds not to the "quantization of the energy" of the system. Since the fields are generated by conserved elementary charges of the elementary particles, the mutual fields can be supposed to be composed by two-particle interactions and at steady-states, the interacting fields are depending only on $\mathbf{r'}_{l,m} = \mathbf{r}_l - \mathbf{r}_m$ and on $\mathbf{v'}_{l,m} = \mathbf{v}_l - \mathbf{v}_m$. The mutual fields are also depending on the relative positions of the different particles and on their relative velocities. Since we don't have any precise information about the positions and the velocities of the particles, we have to use the temporary stationary probability densities

$j'_i{}^{(n)\nu}(\mathbf{r}) = (c \cdot \rho'_i(\mathbf{r}), \mathbf{j'}_i(\mathbf{r}))$, i = e,p,P,E.

In the classical physics based on the knowledge of precise conditions for the positions and for the velocities at each time point, we know how the stationary electromagnetic force, $\mathbf{E}^{(em)}(\mathbf{r})$ and $\mathbf{B}^{(em)}(\mathbf{r})$, is acting on a charge q_i at a position $\mathbf{r}$; that is the so called Lorentz force

$d(m_i \mathbf{v})/dt = \mathbf{F}^{(em)}(\mathbf{r}) = + q_i \cdot (\mathbf{E}^{(em)}(\mathbf{r}) + \mathbf{v}/c x \mathbf{B}^{(em)}(\mathbf{r}))$.

A similar stationary gravitational force is also acting on the elementary gravitational charge g_i

$d(m_i \mathbf{v})/dt = \mathbf{F}^{(g)}(\mathbf{r}) = - g_i \cdot (\mathbf{E}^{(g)}(\mathbf{r}) + \mathbf{v}/c x \mathbf{B}^{(g)}(\mathbf{r}))$.

One problem arises because while the static electric force and the static gravitation forces caused by the elementary charges, q_l and g_l, can be expressed as

$\mathbf{E}^{(em)}(\mathbf{r}) = + q_l \cdot (\mathbf{r} - \mathbf{r}_l)/4\pi|(\mathbf{r} - \mathbf{r}_l)|^3$,

$\mathbf{E}^{(g)}(\mathbf{r}) = + g_l \cdot (\mathbf{r} - \mathbf{r}_l)/4\pi|(\mathbf{r} - \mathbf{r}_l)|^3$,

and depending only on the relative distance, $\mathbf{r} - \mathbf{r}_l$, the magnetic components, $\mathbf{B}^{(em)}(\mathbf{r})$ and $\mathbf{B}^{(g)}(\mathbf{r})$ depend also on the relative velocity, $\mathbf{v} - \mathbf{v}_l$, of the moving charges, q_l and g_l. Generally, the expressions of the vector-fields, $A^{(em)\nu}(x)$ with $\mathbf{E}^{(em)}(\mathbf{r},t)$ and $\mathbf{B}^{(em)}(\mathbf{r},t)$ and $A^{(g)\nu}(x)$ with $\mathbf{E}^{(g)}(\mathbf{r},t)$ and $\mathbf{B}^{(g)}(\mathbf{r},t)$ are the same. In order to get a temporary stationary solution in the classical case, we would have to summarize all the Lorentz forces between all particles in COM at rest and remark that the particles don't leave a finite volume V and no other particles and no radiations go out and come in. Since the magnetic parts are proportional to $(\mathbf{v} - \mathbf{v}_l) x \mathbf{B}^{(em)}(\mathbf{r})$ and $(\mathbf{v} - \mathbf{v}_l) x \mathbf{B}^{(g)}(\mathbf{r})$, the energies of the steady-states are conserved.

Now, I consider special N-particle systems which are electric neutral. Such systems are for instance electric neutral atoms and the two-particle systems with different electric charges, (e,P), (e,p), (P,E) and (p,E). If we suppose that eltons are not involved in the neutral atoms, the atom is composed of A protons, N_p positrons and $(A + N_p)$ electrons with a total number of $N = 2\cdot(A + N_p)$ particles. At first, I should emphasize that at neutral atoms Lagrange multipliers with different values play a role. The Lagrange multiplier, $\lambda_k' = h$, with the value

$$h = 4.135\ 667\ 662\cdot 10^{-15}\ \text{eV}\cdot\text{s}$$

of the Planck constant is fixing the electron shells of the atoms with Z electrons while another Lagrange multiplier, $\lambda_k' = h^0$, with the value

$$h^0 = h/387$$

governs the remaining $2\cdot A - Z + 2\cdot N_p$ particles in the nuclei http://atomsz.com/wp-content/uploads/Variationsprinzip.pdf . I notice, the neutron is a composite particle: it is either N^0 = (P,e), or N = (P,e,p,e). The second type of neutron is instable if it is free. The bound energies of atoms, E(N;bound), are stored in the electron shells and in the nuclei. The electron shells are in the range up to some 10^{-8} cm, the nuclei are in the range of 10^{-13} cm. The principle calculation of the structure of the electron shells and of the structure of nuclei is very similar; one has only to use either h, or h^0. Another physical difference is that while in the electron shells are only electrons and they repulse each other and are attracted by the nuclei, in the nuclei are particles, P, e, p, present which are either repulsive, or attract each other.

An electric neutral atom composed of P, e, and p has the gravitational mass

$$m^g(N) = A\cdot(m_P - m_e)$$

and the inertial mass

$$m^i(N) = A\cdot(m_P + m_e) + 2\ N_p\cdot m_e - E(N;\text{bound})/c^2 \geq 0.$$

The numbers of positrons, $N_{p,}$ in electric neutral atoms with the mass number A are a piori unknown. The gravitational mass and the inertial mass of atoms are obviously different. In the atoms, the bound energies, E(N;bound), depend on two Lagrange multipliers, $\lambda_k' = h$ and $\lambda_k' = h^0$. Generally, a body composed on $N = N_P + N_E + N_p + N_e$ elementary particles e, p, P and E has the gravitational mass

$$m^g(N) = |(N_P - N_E)\cdot m_P + (N_p - N_e)\cdot m_e|$$

and the inertial mass

$$m^i(N) = (N_P + N_E)\cdot m_P + (N_p + N_e)\cdot m_e - E(N;\ \text{bound})/c^2 \geq 0.$$

Galileo's UFF hypothesis is violated https://www.youtube.com/watch?v=WsyJjxC7SRc. The last two equations can be used for the calculation of both masses of all observed composite particles. Einstein's guess within the theory of special relativity that the inertia of bodies are equal to their energy contents, expressed by the mass-energy equivalence relation, $E = m \cdot c^2$, is obviously invalid. The masses of instable composed particles such as for instant the mass of the instable neutron N = (P,e,p,e) can also be calculated with $\lambda_k' = h^0$. However, in these cases the flow of particles from a finite region Ω is different from zero. The flow of particles from Ω can be characterized with a second constant, Γ, which describe the average lifetime of the instable particle. Instable particles are never elementary particles. The excited states of atoms are also instable states having lifetimes, $\Gamma \approx 10^{-7}$ s and lost continuous energy through radiations of electromagnetic field until the ground states of electron shells are reached. Einstein's heuristic assumption of light quanta with the energies, $E = h \cdot \nu$, is not needed.

Two-particle states

The study of two-particle states are interesting because the interactions between $N = N_P + N_E + N_p + N_e$ elementary particles can be considered as composed of two-particle interactions. Among the two-particle states the electric neutral particle systems, (e,P), (e,p), (p,E) and (P,E) are of special interests then only they can build bound states. At first, I write down the gravitational masses and inertial masses of electric neutral two-particle systems:

The gravitational masses: $m^g = (m_P – m_e)$ for (e,P) and (p,E) and

$m^g = 0$ for (e,p) and (P,E).

The inertial masses: $m^i = (m_P + m_e) – E(\text{bound})/c^2$ for (e,P), and (p,E) and

$m^i = 2 \cdot m_e - E(\text{bound})/c^2$ for (e,p) and $m^i = 2 \cdot m_P - E(\text{bound})/c^2$ for (P,E).

We see, the gravitational masses are conserved masses since they are composed on the invariant masses, m_P and m_e. The inertial masses change depending on the value of λ_k'. With $\lambda_k' = h$ the (e,P) system builds the ground state of hydrogen atom, the (p,E) system the ground state of the so called "anti-hydrogen atom", the (e,p) system the ground state of the positronium. The (P,E) system can also build a ground state of protonium, but it was not until 2006 that scientists realized protonium generated during the experiment. Fortunately, the bound energies for all mentioned electric neutral two-particle ground states can be calculated with the known value of the Planck constant, $h = 4.136 \cdot 10^{-15}$ eV·s and with the reduced masses, $m_{ij}' = m_i\, m_j/(m_i + m_j)$, i,j = e,p,P,E, and with $m_e = m_p$ and $m_P = m_E$, according to

$E(h;\text{bound}) = h^2\, m_{ij}' e^4/8.$

An appropriate formula for the Planck constant was already discovered by Arnold Sommerfeld at the beginning of the 20th century which connects the ground state energy of a hydrogen atom, E(bound) = 13.6 eV, with h

$h = e^2/2c \cdot (m_{eP}' \cdot c^2/2 \cdot E(\text{bound}))^{1/2}.$

I generalize this formula to all possible values of Lagrange multipliers

$\lambda_k' = e^2/2c\cdot(m_{ij}'\cdot c^2/2\cdot E(\lambda_k';bound))^{1/2}$.

The ground state energy of the "anti-hydrogen atom" is the same as the ground state energy of the hydrogen atom, while the ground state energy of the positronium is 6.8 eV with the reduced mass $m_{ep}' = m_e/2$. The ground state energy of the protonium is at the energy 12.484 KeV because the reduced mass is $m_{P,E}' = m_{P/2}$ is 1836.1 times greater than that of positronium.

With the known value of h and with the reduced masses, the radii of the two-particle ground states can also be calculated according to the well known formula of the quantum mechanics

$r = h^2/(4\cdot\pi^2\cdot m_{ij}'\cdot e^2)$.

With this formula the radii of ground states of the hydrogen atom, of the co called "anti-hydrogen atom", of the positronium and of the protonium can be calculated to the values of $0.529\cdot 10^{-8}$ cm, $0.529\cdot 10^{-8}$ cm, $1.058\cdot 10^{-8}$ cm and $0.763\cdot 10^{-12}$ cm. Since in the hydrogen atom the proton resides in the middle of the atom its size is the double of its radius, $1.058\cdot 10^{-8}$ cm.

I generalize also the formula for the radii to different valued Lagrange multipliers, λ_k',

$r = \lambda_k'^2/(4\cdot\pi^2\cdot m_{ij}'\cdot e^2)$.

Until now, I have used the notation "ground state" in accordance to the quantum mechanics, in a theory which knows only the Planck constant h. In the atomistic theory of matter I reckon with different values of Lagrange multipliers, λ_k', which lead to lower energetic level than h does. I future, it would be more appropriate to denote with ground states the lowest energy state of a system. I think, it does not lead to any confusion to use another designation of "ground states" in the atomistic theory of matter then in the usual quantum mechanics. In the atomistic theory the stable atoms cannot be regarded as ground states of N particle systems.

The determination of h^0

For the determination of $\lambda_k' = h^0$, I use the two particle systems (e,p) and (P,E). The condition that the inertial masses are zero,

$m^i = 2\cdot m_e - E((e,p);bound)/c^2 = 0$ for (e,p) and

$m^i = 2\cdot m_P - E((P,E);bound)/c^2 = 0$ for (P,E),

are leading to the bound energies

$E((e,p);bound) = 2\cdot m_e\cdot c^2$ and

$E((P,E);bound) = 2\cdot m_P\cdot c^2$.

Form the formula

$\lambda_k' = e^2/2c\cdot(m_{ij}'\cdot c^2/2\cdot E(bound))^{1/2}$

I have got the same value for the Lagrange multiplier, λ_k', in both cases

$\lambda_k' = h^0 = e^2/2c\cdot(1/8)^{1/2} = h/387$.

With this value of λ_k' the radii of these two particle systems, (e,p) and (p,E), are

$r_{(e,p)} = 0.703\cdot10^{-13}$ cm and $r_{(P,E)} = 0.383\cdot10^{-16}$ cm.

We see, the composing particles, (e,p) and (p,E), cannot approach closer to each other than these relative distances. I identify theses states as electron-neutrino, ν_e =(e,p), and proton-neutrino, ν_P =(P,E). In particular, these particles cannot annihilate. With other words, the particles cannot vanish with only generation of energy, such as Einstein guessed in his mass-energy equivalence, $E = m\cdot c^2$. The main condition of the atomistic theory of matter, that the particle numbers, N_i, are conserved, is fulfilled. Additionally, I remark that the particles can also not be generated by the fields. **The atomistic theory of matter based on point-like indivisible elementary particles, e, p, P and E, gives a complete new basic of physics as the nowadays used quantum theories with $E = m\cdot c^2$ accompanied by Einstein's general theory of relativity. The fundamental interactions appear without singularities.**

With $\lambda_k' = h^0$ the stable neutron, N^0 = (e,P) can also be calculated with the bound energy $E(N^0;\text{bound})$ = 2.04 MeV and the size $d = 2\cdot r_{(e,P)} = 0.702\cdot10^{-13}$ cm. Since this is also the size of the electron-neutrino, ν_e =(e,p), we conclude, the nuclei are composed on protons, electrons and positrons and are governed by h^0. The temporary embedded eltons would fall out of the nuclei because the size of the proton-neutrino, ν_P =(E,P), is ca. 2000 times smaller as the size of the nuclei, $d_{nuclei} \geq 10^{-13}$ cm.

Supplementing that an electron + proton system has its ground state at the bound energy

$E((e,P);\text{bound}) = (m_e + m_P)\cdot c^2$,

in the relative distance between e and P which is greater than 10^{-17} cm.

In a two-particle system the relative velocity of bound particles is given by formula

$(v/c)/(1- (v/c)^2)^{1/2} = (2\cdot E(\text{bound})/m_{ij}\cdot c^2))^{1/2}$,

and the relative velocity of particles in the bound states, v, cannot reach c.

Summary: The atomistic theory of matter based on four kinds of point-like indivisible elementary particles e, p, P, and E which carry two kinds of conserved elementary charges, $q_i = \{\pm e\}$ and $q_i = \{\pm g\cdot m_e, \pm g\cdot m_P\}$ is a unified theory of electromagnetism and gravitation with a much simpler theoretical structure as the nowadays accepted theories. But, only if all experimental observation could be described by this theory, thus without quantization of the energy, the mass-energy equivalence, the introduction of weak- and strong-interactions, the quark model and without any ad hoc new physical hypothesis, would we accept the atomistic theory of matter as a unified description of the universe.

* Gyula I. Szász, Ingelheim, Germany, on January 21, 2017