# The Equation of Motion in the Gravitation Field According to Newton's 2. Axiom and Expressed with Gravitational Charges 

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It is easy to drive. Newton's second axiom (the dynamics) sates
Force $=$ mass x acceleration

$$
\begin{equation*}
\mathrm{F}=\mathrm{m}^{\mathrm{i}} \mathrm{a} \tag{1}
\end{equation*}
$$

The mass here is the inertial mass, $\mathbf{m}^{\mathbf{i}}$. The force can be any force. It is important that the inertial mass is related to the 2 . axiom.

For the force on a body with the gravitational mass, $\mathrm{m}^{\mathrm{g}}$, in the gravitational field of a second body with the gravitational mass $M^{g}$ Newton has set up the following expression
$\mathrm{F}=-\mathrm{GM}^{\mathrm{g}} \mathrm{m}^{\mathrm{g}} / \mathrm{r}^{2}$,
where G is the universal gravitational constant and r is the relative distance between both bodies. Inserting this expression in the 2 . axiom, one obtains the equation of motion of the body
$\mathrm{m}^{\mathrm{i}} \mathrm{a}=-\mathrm{GM}^{\mathrm{g}} \mathrm{m}^{\mathrm{g}} / \mathrm{r}^{2}$.
It is important to notice that the inertial mass stays on the left and the gravitational mass on the right side of the equation of motion. A priori, one has to deal with two kinds of masses of each body, $\mathrm{m}^{\mathrm{i}}$ and $\mathrm{m}^{\mathrm{g}}$.

Actually, a vector equation describes the vector force $\mathbf{F}$ and the acceleration $\mathbf{a}$, therefore
$\mathbf{F}=\mathrm{m}^{\mathrm{i}} \mathbf{a}, \mathbf{F}=-\mathrm{GM}^{\mathrm{g}} \mathrm{m}^{\mathrm{g}} \mathbf{r} / \mathrm{r}^{3} \rightarrow \mathrm{~m}^{\mathrm{i}} \mathbf{a}=-\mathrm{GM}^{\mathrm{g}} \mathrm{m}^{\mathrm{g}} \mathbf{r} / \mathrm{r}^{3}$.
The masses, $\mathrm{m}^{\mathrm{i}}$ and $\mathrm{m}^{\mathrm{g}}$, are scalar quantities and the 2 . axiom implies also, that the acceleration points in direction of the force.

Galileo's UFF hypothesis from the 17. century states that all bodies have a universal acceleration, $\mathbf{a}_{0}$, in the gravitational field, that means $\mathbf{a}=\mathbf{a}_{\mathbf{0}}$. According this, the measured values of the gravity acceleration, $\mathbf{a}$, is independent of the properties of bodies. This is only true if for all bodies the inertial and gravitational masses are equal. Then, we would have to deal in physics only with one kind of mass of bodies (and the established physics deals actually only with one kind of mass for each body)
$m^{i}=m^{g}=m$.
Now, according to an ansatz of me, I write the gravity force in Eq. (4) in the following way
$m^{i} \mathbf{a}=-G^{g} m^{g} \mathbf{r} / \mathrm{r}^{3}=-\left(\mathrm{g} \mathrm{M}^{\mathrm{g}}\right)\left(\mathrm{g} \mathrm{m}^{\mathrm{g}}\right) \mathbf{r} / 4 \pi \mathrm{r}^{3}$.
Principally, I have done nothing else then writing the universal gravitational constant as a product
$\mathrm{G}=\mathrm{g}^{2} / 4 \pi$.
I have introduced an additional factor $4 \pi$ because I thought on a surface integral which enclose a volume V in which the second body with the gravitational mass, $\mathrm{M}^{\mathrm{g}}$, is located. In back of my head, I think on two kinds of conserved gravitational charges
$g(M)= \pm \mathrm{g} \mathrm{M}^{\mathrm{g}}$ und $\mathrm{g}(\mathrm{m})= \pm \mathrm{g} \mathrm{m}^{\mathrm{g}}$.
The signs of the gravitational charges are not fixed by Newton's ansatz for the gravitational force, Eq. (2). Only that both gravitational charges, $g(M)$ and $g(m)$, must have the same sign. Only in this chase is be the gravity attractive.

## In the following, I will not consider that the gravitational mass and inertial mass are equal, instead I write

$\mathrm{m}^{\mathrm{g}}($ body $)=\mathrm{m}^{\mathrm{i}}($ body $)(1+\Delta($ body $))$,
The $\Delta$ (body) is called the relative mass defect of a body and at the moment it is defined according to this equation.

With this ansatz for the relation of gravitational and inertial masses and using $G=g^{2} / 4 \pi$ in Eq. (4) and expressing $\mathbf{a}$ (body)
$\mathbf{a}($ body $)=-\mathrm{m}^{\mathrm{g}}($ body $) / \mathrm{m}^{\mathrm{i}}($ body $) \mathrm{G} \mathrm{M}^{\mathrm{g}} \mathbf{r} / \mathrm{r}^{3}$
$=-\mathrm{g}^{2} / 4 \pi \mathrm{M}^{\mathrm{g}} \mathbf{r} / \mathrm{r}^{3}(1+\Delta(\operatorname{body}))=-\mathbf{a}_{0}(1+\Delta($ body $))$,
I get at the end
$\mathbf{a}($ body $)=-\mathbf{a}_{0}(1+\Delta($ body $))$.
In words: The acceleration in the gravitational field of a body, a(body), is connected by the factor $(1+\Delta$ (body)) to the properties of the body with masses $\mathrm{m}^{\mathrm{g}}$ and $\mathrm{m}^{\mathrm{i}} . \mathbf{a}_{0}=\mathrm{g}^{2} / 4 \pi \mathrm{M}^{\mathrm{g}} \mathbf{r} / \mathrm{r}^{3}$ is independent of any properties of the body.

For the following consideration it is important to know that phenomenological (beginning with the mass spectrum experiments of F. Soddy and F. W. Aston in 1920) the inertial masses of all isotopes with the mass number A and with the nuclear charge $Z, \mathrm{~m}^{\mathrm{i}}(\mathrm{A}, \mathrm{Z}$ isotope), are known. If one knows the isotopic composition of a body than the inertial mass of a body can be written as
$\mathrm{m}^{\mathrm{i}}(\mathrm{body})=\Sigma \mathrm{h}(\mathrm{A}, \mathrm{Z}) \mathrm{m}^{\mathrm{i}}(\mathrm{A}, \mathrm{Z}$ isotope $)$.
$h(A, Z)$ is the frequency of occurrence of isotopes with $A$ and $Z$ in the body. One makes a small error here, which is given by the fact that one neglects the bindings of the isotopes in the body. But, this error is very small compared to the bindings of particles within the isotopes. Thus, physics can principally calculate the inertial mass, $\mathrm{m}^{\mathrm{i}}$ (body), form the phenomenological known inertial masse of the isotopes, $\mathrm{m}^{\mathrm{i}}$ (A,Z isotope). But, the established physics doesn't know how to calculate the gravitational mass of bodies $\mathrm{m}^{\mathrm{g}}$ (body), also physics doesn't know how to calculate $\mathrm{m}^{\mathrm{g}}$ (A isotope), and from them the $\mathrm{m}^{\mathrm{g}}$ (body). Therefore, physics cannot calculate the relative mass defect, $\Delta$ (body) and uses always $\Delta$ (body) $=0$. The established physics is considering the gravitational mass always equal to the inertial mass for each body.

Now, I take my atomistic theory of matter, www.atomsz.com, and assume that all bodies are composed of four kinds of point-like stable elementary particles e, p, P and E. Furthermore I assume that these particles carry, besides the conserved electric elementary charges, $\pm \mathrm{e}$, also the conserved elementary gravitational charges
$g($ elementary particle $i)=\left\{ \pm \mathrm{g} \cdot \mathrm{m}_{\mathrm{e}}, \pm \mathrm{g} \cdot \mathrm{m}_{\mathrm{P}}\right\}, \mathrm{i}=\mathrm{e}, \mathrm{p}, \mathrm{P}, \mathrm{E}$
If the signs of the elementary gravitational charges are correctly distributed to the particles, e, p, P and E, then for electric neutral isotopes the gravitational masses are

$$
\begin{equation*}
\mathrm{m}^{\mathrm{g}}(\mathrm{~A} \text { isotope })=\mathrm{A}\left(\mathrm{~m}_{\mathrm{P}}-\mathrm{m}_{\mathrm{e}}\right) . \tag{14}
\end{equation*}
$$

The elton (E) is called "anti-proton" in physics. In Eq. (14) it is assumed that the eltons are not incorporated in the isotopes. Thus, the isotopes are only composed of electrons (p), positrons (p) and protons (P). The sign of the gravitational charge of proton is fixed as positive (it is a convention)
$\mathrm{g}($ proton $)=+\mathrm{g} \mathrm{m}_{\mathrm{p}}$
and the sign of the gravitational charge of the electron is negative,
$\mathrm{g}($ electron $)=-\mathrm{g} \mathrm{m} \mathrm{e}_{\mathrm{e}}$.
The number of positrons, $N_{p}$, doesn't occur in $m^{g}(A$ isotope $)$ since it is
$\mathrm{g}($ positron $)=+\mathrm{g} \mathrm{m}_{\mathrm{e}}$,
and therefore, in the electric neutral isotopes the number $\mathrm{N}_{\mathrm{p}}$ of positrons is neutralized by additional number $\mathrm{N}_{\mathrm{p}}$ of electrons, also regarding the gravitational charges. The gravitational masses of isotopes, $\mathrm{m}^{\mathrm{g}}$ ( A isotope) depend only on A and are a multiple of $m_{P}-m_{e}$.

I conclude: only in the atomistic theory of matter, gravitational masses of isotopes can be calculated according to Eq. (14) and therefore, the gravitational mass of a body is
$\mathrm{m}^{\mathrm{g}}($ body $)=\Sigma \mathrm{h}(\mathrm{A}) \mathrm{m}^{\mathrm{g}}(\mathrm{A}$ isotope $)=\mathrm{N}\left(\mathrm{m}_{\mathrm{P}}-\mathrm{m}_{\mathrm{e}}\right)$.
N is the number of protons in the body. Since the inertial masses of all isotopes, $\mathrm{m}^{\mathrm{i}}$ (A,Z isotope), are phenomenological known, the $\Delta$ (body) can be calculated for any composition. It turns out to be
$-0.109 \%$ (hydrogen atom) $<\Delta$ (body) $<+0.784 \%\left({ }^{56} \mathrm{Fe}\right.$ isotope),
The calculation of $\Delta$ (body) is very easy if the body is composed of only one kind of isotope
$\Delta($ body $)=\Delta(\mathrm{A}, \mathrm{Z}$ isotope $)=\mathrm{A}\left(\mathrm{m}_{\mathrm{p}}-\mathrm{m}_{\mathrm{e}}\right) / \mathrm{m}^{\mathrm{i}}(\mathrm{A}, \mathrm{Z}$ isotope $)-1$.
I have calculated $\Delta(\mathrm{A}, \mathrm{Z}$ isotope) for the most frequently occurring isotopes of each chemical element, and so I know how the UFF-hypothesis is violated for diverse bodies. The planetary orbits do also not confirm the UFF. It is not $\mathbf{R}_{\mathrm{j}}{ }^{3} / \mathrm{T}_{\mathrm{j}}{ }^{2}=$ const, as the third law of Kepler requires, https://www.youtube.com/watch?v=WsyJjxC7SRc.. The observed violation of the UFF hypothesis is a confirmation of the atomistic theory of matter, in contrast to the established physics which uses exclusively $m^{i}=m^{g}$ and energetic aspects. The stable elementary particles $e, p, P$ and $E$ can neither be annihilated, not generated. The pair of elementary particles ( $e, P$ ) and ( $\mathrm{P}, \mathrm{E}$ ) are forming two kind of basic neutrinos.

The phenomenological calculation of $\Delta$ (body) does not require the following, but for the sake of completeness, in the atomistic theory, also the inertial masses of bodies (and therefore also for the isotopes), composed of $\mathrm{N}_{\mathrm{i}}$ elementary particles, $i=e, p, P, E$, can be calculated
$\mathrm{m}^{\mathrm{i}}($ body $)=\left(\mathrm{N}_{\mathrm{P}}+\mathrm{N}_{\mathrm{E}}\right) \mathrm{m}_{\mathrm{P}}+\left(\mathrm{N}_{\mathrm{p}}+\mathrm{N}_{\mathrm{e}}\right) \mathrm{m}_{\mathrm{e}}-\mathrm{E}($ binding $) / \mathrm{c}^{2}>0$.
The energy, E(binding), is radiated when the particles bind in the bodies and this binding energy can be calculated using a variation principle http://atomsz.com/covariant-theory/.

Additionally I note, that in the here presented considerations, the inertial masses can always be understood as the inertial rest masses. From the equation of dynamics follows, that the inertial mass is a function of the velocity of the body compared to the velocity of light, v/c. Generally, c gives the propagation speed of the interactions between particles and this constant speed is independent of the state of motion of the particles. Therefore, the space and time is connected and the Minkowski space incorporates this connection.

The similarity in the structure of electromagnetism and gravity, which begins with the conserved electric an gravitational elementary charges, $\mathrm{q}_{\mathrm{i}}=\{ \pm \mathrm{e}\}$ and $\mathrm{g}_{\mathrm{i}}$ $=\left\{ \pm \mathrm{g} \cdot \mathrm{m}_{\mathrm{e}}, \pm \mathrm{g} \cdot \mathrm{m}_{\mathrm{p}}\right\}$ of the elementary particles, $\mathrm{i}=\mathrm{e}, \mathrm{p}, \mathrm{P}$ and E , is continued in the four vector fields, $A^{(e m) v}(x)$ and $A^{(g) v}(x)$ which propagate with $c$ in finite regions of the Minkowski space, $\{x\} \in \Omega$. Also the connection of the four vector fields to $\mathbf{E}^{(e m)}(\mathrm{x})$ and $\mathbf{B}^{(\mathrm{em})}(\mathrm{x})$ respectively to $\mathbf{E}^{(\mathrm{g})}(\mathrm{x})$ and $\mathbf{B}^{(\mathrm{g})}(\mathrm{x})$ is similar. Also gravitational Lorentz force http://atomsz.com/theory/stable-particles-and-a-unified-field-eng/
$\mathbf{F}_{\text {Lorentz }}{ }^{(\mathrm{g})}(\mathbf{r}, \mathrm{t})=-\mathrm{g}_{\mathrm{i}}\left(\mathbf{E}^{(\mathrm{g})}(\mathrm{x})+(\mathbf{v} / \mathrm{c}) \mathbf{x} \mathbf{B}^{(\mathrm{g})}(\mathrm{x})\right)$
is derived for gravity which can be used if the velocity of bodies are not small to c. The abnormal precession of the orbit of Mercury around the Sun can be calculated with the gravitational Lorentz force. But this calculation has not been performed yet but it would probably give a further confirmation of the atomistic theory. The abnormal precession of Mercury must not be a confirmation of the general relativity which has led to deformation of space and time. In any ways, the electromagnetic field propagates independently of the gravitational field in finite ranges from Minkowski space, apart from the fact that both fields are caused by two kinds of elementary charges of the same elementary particles.

One can also emphasize that Einstein's mass-energy-relation was not right
$\mathrm{E}=\mathrm{m}^{*} \mathrm{c}^{2}$,
because $\mathbf{m}^{*}$ is not the inertial mass, as Einstein claimed 1905, [1], in the frame of his special relativity theory. But, m* is a kind of third mass and only Einstein
knows what mass is it. Definitely in the frame of atomistic theory of matter the equivalence principle of Einstein is disproved: Energy is not equivalent to mass.

Opposite to the given fundaments of the established physics, the new physical axioms are:

Four kinds of point-like stable elementary particles e, $\mathbf{p}, \mathbf{P}$ and $\mathbf{E}$ exist.

- The elementary particles carry two kind of elementary charges, $q_{i}=\{ \pm e\}$ and $g_{i}=\left\{ \pm \mathbf{g} \cdot \mathrm{m}_{\mathrm{e}}, \pm \mathrm{g} \cdot \mathrm{m}_{\mathrm{P}}\right\}, \mathrm{i}=\mathbf{e}, \mathbf{p}, \mathbf{P}, \mathrm{E}$.
- The elementary charges cause the interactions between particles and they cause also the interaction fields.
-The interactions propagate with $c$ and the constant propagation is independent of the state of the motion of particles.

Because of the physical measurements, it should be taken into account

- that measurements with infinite precision cannot be assumed,
- each measurement is performed in finite regions of space and time.
[1] Albert Einstein, Ist die Trägheit eines Körpers von seinem Energieinhalt abhängig?, Ann. d. Phys., 17, 891 (1905)

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