7. Treatment of the Fundamental Field with Calculus of Variations

Abstract: A fundamental revision of the accepted Standard Model of Physics has to be performed because neither the assumption of classical physics, the equivalence of inertial mass and gravitational mass, nor the used quantum conditions of microscopic physics appear to be valid hypotheses. The fundamental field (UF) consisting of the electromagnetic field and the covariant gravitational field generated by four types of sources is central in physics. The sources = quanta of the UF are represented through the four stable particles e, p, P and E, having two kinds of Maxwell charges. The source quantization leads to variation principles of open physical systems in finite space-time domains Ω. Beside Planck's constant h, at least a second basic constant $h^0 = 1/(4\sqrt{2}) \times q^2/c$ exists which is responsible for the neutrinos as bound states (e,p), (P,E) and for nuclear forces. The source quantization of the UF is resulting in investigations of variation problems within a set of new fundamental hypotheses of physics.
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The accepted Standard Model seems precisely explain nature. However, criticism has to be made on the historically gathered hypotheses which is not marginal but fundamental. The result leads to the manifest covariant fundamental field (UF) consisting of electromagnetism and gravity generated through the four stable particles with two kinds of Maxwell charges. The composition dependent relation

$(m^g - m^i) = \Delta E^{bound}/c^2 > 0,$

of the inertial and gravitational mass of atoms is verified by the author in a fall experiment in Ref. **[6],** see also Refs. **[1, 2]**. Furthermore, all microscopic objects are essentially smaller than the characteristic wave lengths of their radiation **Fig. 1**. Therefore, a dominance of the wave character of the electromagnetic field can be assumed in all microscopic processes. The explanation of the line spectra is connected the source quantization, with the usage of open systems and with restrictions of physical descriptions onto finite space-time domains, Refs. **[3, 4]**. A set of new basic principles of physics, Ref. **[5]**, leads to variation problems with isoperimetric subsidiary and natural boundary conditions. Planck's constant h and a second fundamental constant h^0 are supposed to be connected with Lagrange multipliers. The latter constant is responsible for the neutrinos and for the nuclear forces, Ref. **[3]**. The assumed universality of h and of Heisenberg's uncertainty relation appear as impermissible generalizations. The new basic hypotheses flow in a New Model**,** centralizing in physics the Unified Field (UF) and the four stable point-like elementary particles with two Maxwell charges: the electron (e), positron (p), proton (P) and the negative charged proton with the name elton (E). These four particles quantize the sources of the UF and are able to elucidate many features nowadays unknown.

In order to avoid discrepancies and impermissible generalizations, which are inherent in the accepted assumptions of physics, and to clear up principal unknown, a set of new basic hypotheses is formulated by the author in Ref. **[5]**. The basic set contains seven conclusive and most probably complete assumptions with axiomatic character.

1.) Basic restrictions on the physical descriptions**:** The physical description of nature is limited onto finite domains of space and time and all physical systems are open systems**.**
2.) One fundamental interaction field exists with unified propagation**:** Only one frame invariant fundamental interaction exists consisting of the electromagnetic field and the covariant gravitational field. The field propagation is finite, constant and has the value c in each frame, see Ref. **[9].** This field is the Unified Field**.**
3.) The elementary particles (EP)**:** Only four types of stable point-like and structure-less particles exist with two Maxwell charges. These are the particles, e, p, P and E.
4.) The finite and invariant c defines unique distances. The property of the space-time continuum**:** In finite space-time domains, space and time is homogeneous, the space is isotropic. A unique Riemann's type metric exists determined uniquely by the UF in a finite domain of the (3, 1) dimensional space-time continuum. The unique metric between particles is given by the propagation constant c of the UF. The invariant infinitesimal distance ds is defined by

$$(ds)^2 = dx_\alpha dx^\alpha = (dx_1^2 + dx_2^2 + dx_3^2) - (cdt)^2 . \quad (1)$$

Additional fundamental assumptions**:**
5.) There exists a separation principle for single elementary particles in very small and for many particle systems in very large space-time distances.
6.) The canonical coordinates of elementary particles are principally undeterminable. This hypothesis is more general than Heisenberg's uncertainty relation with h.
7.) The two kinds of charges, the e-charges as well as the g-charges, have two signs. The amount of the e-charge, q, is the same for all particles. But the amount of the g-charges is only equal for e and p respectively for P and E. The amount of g-charge g_i is proportional to the rest mass m_e and m_P of an elementary particle: electron: $g_1 = -gm_e$, positron: $g_2 = +gm_e$, proton: $g_3 = +gm_P$, elton: $g_4 = -gm_P$. We shall use this index convention of EP here. The universal gravitational constant is $\mathbf{G} = g^2/4\pi$.

The set of basic hypotheses leads to a New Model of nature in which few of the assumptions gathered over time, remain valid: only the existence of atoms, of elementary electric charges and of the propagation of light c. The New Model and the accepted Standard Model are essentially different and they give a very controversial explanation of nature. A crucial difference is the explanation of the observed gravity generated through the gravitational charges of the four EP, Ref. **[2]**. Furthermore, the Eikonal theory prohibits the existence of photons in microscopic reactions, Ref. **[4, 5]**.

Here we want to continue the investigation of the New Model**.** It is not only a mathematically correct theory based on a few conclusive hypotheses but also the laws of nature are represented within it. The electromagnetic field is described with the known equations

$$\partial_\alpha \partial^\alpha A^{(e)\ \beta} = + j^{(e)\ \beta}; \ A^{(e)\ \beta} = (\phi^{(e)}/c, \mathbf{A}^{(e)}), \quad (2a)$$

where

$$j^{(e)\ \beta} = (\rho^{(e)}, \mathbf{j}^{(e)}/c)$$

is used with

$\partial_\beta j^{(e)\beta} = 0$, the continuity equation and e-charge conservation, (2b)

and

$\partial_\beta A^{(e)\beta} = 0$, the Lorentz gauge, the conservation of the e-field properties. (2c)

In complete analogy to electromagnetism, the covariant gravity is described in terms of four-vector potential $A^{(g)\beta}$ and of four-current $j^{(g)\beta}$ in a finite space-time domain Ω, according to the equations

$\partial_\alpha \partial^\alpha A^{(g)\beta} = -j^{(g)\beta}$; $A^{(g)\beta} = (\phi^{(g)}/c, \mathbf{A}^{(g)})$, (3a)

where $j^{(g)\beta} = (\rho^{(g)}, \mathbf{j}^{(g)}/c)$

is used with

$\partial_\beta j^{(g)\beta} = 0$, the continuity equation and g-charge conservation, (3b)

and

$\partial_\beta A^{(g)\beta} = 0$, the Lorentz gauge, the conservation of the g-field properties. (3c)

According to (2a) and (3a) all moving bodies radiate electromagnetic and gravitational rays. The e-field and g-field is connected with the invariant elementary charges of the four stable particles. The charges are defined by surface integrals of the static fields whereby the surface S encloses a small finite volume V containing only one EP. The charges of the four EP are

$\oint_S \mathbf{E}_i^{(e)}.d\mathbf{s} = +q_i$ with $q_i = \{-q, +q, +q, -q\}$, i = 1, 4 and (4a)

$\oint_S \mathbf{E}_i^{(g)}.d\mathbf{s} = -g_i$ with $g_i = \{-gm_e, +gm_e, +gm_P, -gm_P\}$, i = 1, 4. (4b)

With the fixing of the signs of the elementary charges q_i, g_i and the fields,

$\mathbf{E}_i^{(e)} \equiv \mathbf{F}^{Coulomb}/q_j = +q_i \mathbf{r}/(4\pi r^3)$

and

$\mathbf{E}_i^{(g)} \equiv \mathbf{F}^{Newton}/g_j = -g_i \mathbf{r}/(4\pi r^3)$

and with a positive sign convention for g-charge of the proton, the relation

$g_3 = +gm_p$,

is used. The static Coulomb force $\mathbf{F}^{Coulomb}$ and Newton force $\mathbf{F}^{Newton}$ are equivalent with the static zero components of Eqs. (2a) and (3a). The gravitational field $\mathbf{E}_i^{(g)}$ is directed towards the proton with a positive g-charge.

The stress-energy tensor of the gravitational field, $F^{(g)\lambda\rho}$, can be expressed with the components of the four-vector potential $A^{(g)\beta}$ for a Lagrangian in complete analogy to $F^{(e)\lambda\rho}$ expressed with $A^{(e)\beta}$. As a consequence of the two kinds of elementary charges of the four EP, the $A^{(g)\beta}$ and $A^{(e)\beta}$ must always be added in the UF Theory. Due to the weakness of the gravity, the influence of $A^{(g)\beta}$ on the EP can be studied only experimentally with electric neutral particle systems. In a finite volume V enclosed by a surface S such systems are the four basic two-particle systems (e,P), (p,E), (e,p) and (P,E) and we want to investigate mainly these four kinds of two-particle systems here. The net g-charges of these systems have the values

$g_{(e,P)} = +g(m_P - m_e)$, $g_{(p,E)} = -g(m_P - m_e)$, $g_{(e,p)} = 0$ and $g_{(P,E)} = 0$. (5)

The corresponding gravitational masses of the first two systems are

$m^g = m_P - m_e$.

They are the (e,P) and (p,E) systems. The other two systems have formally

$m^g = 0$.
Because of zero gravitational mass, the bound systems (e,p) and (P,E) are "mass-less". They will be identified with two kinds of basic neutrinos, the electron-neutrino υ_e = (e,p) and proton-neutrino υ_P = (P,E).

The static electric force, $\mathbf{F}^{(e)}$, between two net electric charges q_1 and q_2 are given by Coulomb's law. The static gravitational force, $\mathbf{F}^{(g)}$ between two net gravitational charges g_1 and g_2 is defined by Newton's law. For finite relative distances *r* we have

$$\mathbf{F}^{(e)} = +\frac{1}{4\pi}\frac{q_1 q_2}{r^3}\mathbf{r}, \tag{6a}$$

$$\mathbf{F}^{(g)} = -\frac{1}{4\pi}\frac{g_1 g_2}{r^3}\mathbf{r} = \mp\, \boldsymbol{G}\frac{m_1^g m_2^g}{r^3}\mathbf{r}. \tag{6b}$$

In case of electron, $F^{(e)}$ is about a factor ~ $3\text{x}10^{42}$ greater than $F^{(g)}$. For the above considered four electric neutral two-particle systems only the two systems (e,P) and (p,E) have a non-zero gravitational forces $F^{(g)}$:

$$\mathbf{F}^{(g)}_{(e,P),(e,P)} = \mathbf{F}^{(g)}_{(p,E),(p,E)} = -\,\boldsymbol{G}\frac{(m_P - m_e)^2}{r^3}\mathbf{r}, \tag{7a}$$

$$\mathbf{F}^{(g)}_{(e,P),(p,E)} = +\,\boldsymbol{G}\frac{(m_P - m_e)^2}{r^3}\mathbf{r}. \tag{7b}$$

The two mass-less neutrinos exert neither a static electric nor a static gravitational force to other EP. But the Newtonian force between an (e,P) and a (p,E) system is repulsive. In nature, two kinds of matter condensation seem to existing. The one that condensates only with the particles (e, p, P), this is our "world". The other condensation of matter is with (e, p, E). Between bodies both of this type of condensed matter, a repulsive force is present. Condensation of matter with all four EP does not seem to exist. The dark matter is consisting of free flying neutrinos which can only build small particle aggregates, for instant a ν_{ep} = (P,e,p,E) system. Charged particles split these neutrino aggregates, among them the so called composite-neutrino ν_{ep} into the two basic neutrinos, see the decay of charged Myons. The dark matter is over all present in all experimental equipments. Therefore, the presence of dark matter should be taken into account in the interpretation of all scattering experiments at energies higher then ca. 100 MeV.

Figure 1. A comparison of the sizes of microscopic objects with the smallest wave lengths of their electromagnetic radiations. The ionization by atoms means the ionization of the last electron. The size of the neutrinos, ν_e = (e,p) and ν_p = (P,E), and of the neutron, n^0 = (e,P), is calculated with h^0. The size of the electron is also drawn at ~ 10^{-18} cm, as a limit up to which this particle is supposed to be point-like.

According to the Eikonal theorem, this representation illustrates that the field quantization with photons is prohibited as used in the accepted quantum electrodynamics.

Variation problems for two-particle systems in a finite space-time domain Ω**:** The Eikonal theory prohibits light corpuscle in each microscopic process, see

Fig. 1. Therefore, we must use the quantization of the source of the UF instead of the quantization of the electromagnetic field with photons, see Ref. **[3]**. The static quantum condition for the EP in a finite volume V with

$j_i^{(e)\,\alpha} = q_i j_i^{(n)\,\alpha} = q_i(\rho_i^{(n)}, \mathbf{j}_i^{(n)}/c)$,

issues

$$\int_V \rho_i^{(n)} d^3x = n_i = \text{number of particles i in V, for i = 1, 4.} \qquad (8)$$

The four two-particle systems (e,P), (p,E), (e,p) and (P,E) are corresponding to the combinations (i = 1, j = 3); (i = 2, j = 4); (i = 1, j = 2) and (i = 3 and j = 4) whereby both particles are within the same finite volume V. From the continuity equations of the charges, the continuity equations and conservations of particle numbers follow

$$\partial_\alpha j_i^{(n)\,\alpha} = 0 \rightarrow \qquad (9a)$$

$$\partial_0 \int_V \rho_i^{(n)} d^3x = -\int_V \frac{1}{c} \nabla . \mathbf{j}_i^{(n)} d^3x = -\oint_S \frac{1}{c} \mathbf{j}_i^{(n)} . \mathbf{ds}, \text{ for i = 1, 4.} \qquad (9b)$$

The change of time of particles in finite volume V is given by the negative flow of particles through the surface S enclosing the finite volume V.
The action integral of the covariant Lagrange density

$L = L^T - L^{UF}$,

expressed with $A^{(e)\,\alpha}$, $A^{(g)\,\alpha}$ and with the four-currents $j^{(e)\,\alpha}(x)$ and $j^{(g)\,\alpha}(x)$, is

$$I = \int_\Omega (dx^4)\{L^T(x) - L^{UF}(A^{(e)\,\alpha}(x), A^{(g)\,\alpha}(x), j^{(e)\,\alpha}(x), j^{(g)\,\alpha}(x))\}, \qquad (10)$$

in a finite space-time domain Ω. The Lorentz scalar $L^T(x)$ describes usually the Lagrange density for the kinetic energy of particles. The $j^{(e)\,\alpha}$ and $j^{(g)\,\alpha}$ can be expressed with the two kinds of charges q_i and g_i and with the four-current of particle numbers $j_i^{(n)\,\alpha}$,

$$j^{(e)\,\alpha} = \sum_{i=1,4} q_i j_i^{(n)\,\alpha},$$

$$j^{(g)\,\alpha} = \sum_{i=1,4} g_i j_i^{(n)\,\alpha}.$$

Therefore, the Lagrange density can be represented with

$L^0 = -(F^{(e)}{}_{\lambda\rho} F^{(e)\,\lambda\rho} + F^{(g)}{}_{\lambda\rho} / F^{(g)\,\lambda\rho})/4$,

therefore

$$I = \int_\Omega (dx^4)\{L^T(x) - L^0(A^{(e)}, A^{(g)}) - \sum_{i=1,4} (q_i A^{(e)}{}_\alpha - g_i A^{(g)}{}_\alpha) j_i^{(n)\,\alpha}\}. \qquad (11)$$

The well known variations respectively the field quantities $A^{(e)}$ and $A^{(g)}$ only,

$$\delta|_{A^{(e)}} I = \delta|_{A^{(e)}} \int_\Omega (dx^4)\{L^{UF}\} \qquad (12a)$$

and

$$\delta|_{A^{(g)}} I = \delta|_{A^{(g)}} \int_\Omega (dx^4)\{L^{UF}\}, \qquad (12b)$$

give the field equations (2a) and (3a) for the electromagnetic and gravitational field. The field quantities $A^{(e)}$ and $A^{(g)}$ must fulfil the additional conditions Eqs. (2c), (3c).

Now, we want to perform a variation respective the field quantities of particles which are only contained in $j_i^{(n)\,\alpha}$ and $L^T(x)$. The term L^0 corresponds to the particle free part of the field. Therefore, we leave out L^0 and consider the following expression

$$I^{(n)} = \int_\Omega (dx^4)\{L^T - \sum_{i=1,4} (q_i A^{(e)}{}_\alpha - g_i A^{(g)}{}_\alpha) j_i^{(n)\,\alpha}\}. \tag{13}$$

For the two-particle systems (e,P), (p,E), (e,p) and (P,E), we have the integral

$$I_{i,j}^{(n)} = \int_\Omega (dx^4)\{L_{i,j}^T - (q_i A^{(e)}{}_\alpha - g_i A^{(g)}{}_\alpha)(j_i^{(n)\,\alpha} - j_j^{(n)\,\alpha})\}, \tag{14}$$

for the variation. In case of particles, we have the conditions Eqs. (2b), (3b) and additional Eqs. (8), (9) for the field quantities. Now we know that the contribution of the gravitational field $g_i A^{(g)}$ is in order of ~ 10^{-42} less than the contribution of the electromagnetic field $q_i A^{(e)}$ in Ω. For simplicity only, we neglect the contribution of $g_i A^{(g)}$ in Ω. With

$$j_{i,j}^{(n)\,\alpha} = j_i^{(n)\,\alpha} - j_j^{(n)\,\alpha}$$

the remaining covariant Lagrange function is

$$I_{i,j}^{(e),(n)} = \int_\Omega (dx^4)\{L_{i,j}^T(x) - q_i A^{(e)}{}_\alpha(x) j_{i,j}^{(n)\,\alpha}(x)\}. \tag{15}$$

The equations from (8) to (15) describe the mathematic conditions so far. They are defined in a fixed but arbitrary coordinate system in a finite domain of the Minkowski space. But an absolute coordinate system has no relevance in physics and we must go over to a physical relevant relative coordinate system between the particles, between the sources of the UF. In the two-particle case, such a coordinate system leads to the relative four-vectors of particles, to

$$x^\alpha = x_i^\alpha - x_j^\alpha$$

and to the relative four-momentum

$$p^\alpha = p_i^\alpha - p_j^\alpha.$$

The "center-of-momentum" (C-O-M) system, loosely spoken the "center of mass" system, should be described with two other four-vectors X^α and P^α. The C-O-M system corresponds to a fixed but arbitrary outer coordinate system in Minkowski space. Since such an absolute coordinate system has no relevance to the relative motion, we choose a coordinate system with

$$P^\alpha = (E^*/c, 0, 0, 0),$$

in which the "center of mass" is in rest. Hereby

$$M = E^*/c^2,$$

will be interpreted as the mass of composite particle. In this coordinate system the four-current

$$j_{i,j}^{(n)\,\alpha} \equiv (\rho_{i,j}^{(n)}(x^\alpha), \mathbf{j}_{i,j}^{(n)}(x^\alpha)/c)$$

and $L_{i,j}^T(x^\alpha)$ are functions of x^α. The function $\rho_{i,j}^{(n)}(x^\alpha)$ describes the particle number density and $\mathbf{j}_{i,j}^{(n)}(x^\alpha)$ the particle number current for the two-particle system with the condition that if the particle j has the four-coordinate x_j^α than the particle i has the four-coordinate

$$x^\alpha = x_i^\alpha - x_j^\alpha,$$

and the particle mass is

$m_{ij}' = m_i m_j/(m_i+m_j)$.

This relativistic treatment corresponds to the reduction of the equivalent one body problem of classical mechanics if the interaction potential depends on the relative vector **x** and on the time derivative of **x**. The variation problem for particles requires the usage of variation functions η_i instead of densities $\rho_i^{(n)}$ and currents $\mathbf{j}_i^{(n)}$. The functions η_i, i = 1, 4, are the field quantities of the four type of particles and represent the state of particle systems in a finite domain Ω, see also the 6th basic assumption. We set

$$\rho_{i,j}^{(n)}(x^\alpha) = \eta_{i,j}^*(x^\alpha)\eta_{i,j}(x^\alpha), \tag{16}$$

$$\mathbf{j}_{i,j}^{(n)}(x^\alpha)) = \frac{1}{2i}\frac{h'}{m_{ij}'}(\eta_{i,j}(x^\alpha)\nabla\eta_{i,j}^*(x^\alpha)-\eta_{i,j}^*(x^\alpha)\nabla\eta_{i,j}(x^\alpha)), \tag{17}$$

where $\eta_{i,j}^*$ is the complex conjugate of the scalar function $\eta_{i,j}$, and h' is any appropriate chosen constant. Eqs. (8) and (9) with particle number conservation and with the boundary equation must also be fulfilled:

$$\int_V \rho_{i,j}^{(n)} d^3x = 1, \tag{18a}$$

$$\partial_0 \int_V \rho_{i,j}^{(n)} d^3x = -\int_V \frac{1}{c}\nabla.\mathbf{j}_{i,j}^{(n)} d^3x = -\oint_S \frac{1}{c}\mathbf{j}_{i,j}^{(n)}.\mathbf{ds}. \tag{18b}$$

For the kinetic part of Lagrange density, we try to set the Lorentz scalar expression

$$L_{i,j}^T(x^\alpha) = c^2 h''\partial_\alpha\eta_{i,j}^*(x^\alpha)\partial^\alpha\eta_{i,j}(x^\alpha)- m_{ij}'c^2\eta_{i,j}^*(x^\alpha)\eta_{i,j}(x^\alpha), \tag{19}$$

with a second appropriate constant h''. The chosen expressions for $\rho_{i,j}^{(n)}$, ,$\mathbf{j}_{i,j}^{(n)}$ and $L_{i,j}^T$ give a covariant Lagrange density which contains only $\eta_{i,j}^*$, $\eta_{i,j}$ and the first derivative of these functions and some appropriate chosen constants.

For the action integral, Eq. (15), we have the following Lagrangian

$$I_{i,j}^{(e),(n)} = \int_\Omega (dx^4)\{c^2 h''\nabla\eta_{i,j}^*(x^\alpha)\nabla\eta_{i,j}(x^\alpha)-h''\frac{d}{dt}\eta_{i,j}^*(x^\alpha)\frac{d}{dt}\eta_{i,j}(x^\alpha)$$
$$-m_{ij}'c^2\eta_{i,j}^*(x^\alpha)\eta_{i,j}(x^\alpha)-q_i\phi^{(e)}(x^\alpha)\eta_{i,j}^*(x^\alpha)\eta_{i,j}(x^\alpha)$$
$$+q_i\mathbf{A}^{(e)}(x^\alpha)\frac{1}{2i}\frac{h'}{m_{ij}'}(\eta_{i,j}(x^\alpha)\nabla\eta_{i,j}^*(x^\alpha)-\eta_{i,j}^*(x^\alpha)\nabla\eta_{i,j}(x^\alpha))\}. \tag{20}$$

According to Hamilton principle, the Euler-Lagrange equations for the functions $\eta_{i,j}$ are

$$h''\frac{d^2}{dt^2}\eta_{i,j}(x^\alpha)+m_{ij}'c^2\eta_{i,j}(x^\alpha)+h''c^2\Delta\eta_{i,j}(x^\alpha)+q_i\phi^{(e)}(x^\alpha)\eta_{i,j}(x^\alpha)$$
$$+q_i\mathbf{A}^{(e)}(x^\alpha)\frac{1}{i}\frac{h'}{m_{ij}'}\nabla\eta_{i,j}(x^\alpha) = 0 \text{ for i, j = 1, 4} \tag{21}$$

and the complex conjugate equation for $\eta_{i,j}^*$. In this relativistic covariant equation of motion, the second time derivative of $\eta_{i,j}$ and the magnetic field $\mathbf{A}^{(e)}$ appears. To get a static equation, we may set

$\eta_{i,j}(x^\alpha) = \eta_{i,j}(\mathbf{x})\exp(-iE't\,2\pi/h)$,

with some constant h and obtain

$$(m_{ij}'c^2 - (2\pi/h)^2 E'^2 h'')\eta_{i,j}(\mathbf{x}) + h''c^2 \Delta\eta_{i,j}(\mathbf{x}) + q_i \phi^{(e)}(x^\alpha)\eta_{i,j}(\mathbf{x})$$

$$+q_i \mathbf{A}^{(e)}(x^\alpha)\frac{1}{i}\frac{h'}{m_{ij}'}\nabla\eta_{i,j}(\mathbf{x}) = 0. \qquad (22)$$

If we compare this equation with the non-relativistic stationary equation for the H atom, set up by Erwin Schrödinger in 1926, we must omit the term with the vector potential $\mathbf{A}^{(e)}$ whereby h' can not be determined. Furthermore, we have to set

$$h/2\pi = \hbar,\quad h'' = \hbar^2/(2m_{ij}'c^2),\quad -E = m_{ij}'c^2(1-(E'/m_{ij}'c^2)^2/2), \qquad (23)$$

with h = Planck's constant. Then, we formally get the stationary Schrödinger equation,

$$-E\eta_{i,j}(\mathbf{x}) + \frac{\hbar^2}{2m_{ij}'}\Delta\eta_{i,j}(\mathbf{x}) + q_i\phi^{(e)}(x^\alpha)\eta_{i,j}(\mathbf{x}) = 0, \qquad (24)$$

with eigenvalues $E_k<0$ in which E_k was interpreted as the non relativistic energies of the H atom. In this non-relativistic stationary approach the vector potential $\mathbf{A}^{(e)}$ is omitted, and for the eigenvalues E_k, no radiation is caused by

$$\rho^{(e)}_{i,j;k}(\mathbf{x},t) = q_i\,\eta_{i,j;k}{}^*(\mathbf{x},\mathbf{t})\,\eta_{i,j;k}(\mathbf{x},\mathbf{t}),$$

because $\rho^{(e)}_{i,j;k}(\mathbf{x},t)$ does not depend on the time.

Furthermore, we have to notice the mass of the composite particle M. Since

$$P_\mu P^\mu = E^{*2}/c^2 = M^2c^2 \qquad (25)$$

is considered as an invariant, M must be a constant in each frame. But experimentally a mass defect ΔM is observed for composite particles. The inertial mass is less than the gravitational mass, and the difference corresponds to the binding energy of the system, see Ref. **[6]**. The energy-mass equivalence relation

$$\Delta E = \Delta m^i c^2$$

is valid only for an inertial mass. The gravitational mass m^g is unchangeable. There is a fundamental difference between an inertial mass and a gravitational mass and the accepted base in physics on the usage of mass must be revised. The inertial mass of composite particle systems M is an inappropriate quantity in order to use it in particle processes.

The mathematical problem of the source quantization of the UF in a finite volume V is to seek a solution η_{ij} on the isoperimetric subsidiary and natural boundary conditions

$$\int_V \rho^{(n)}_{i,j}(\mathbf{x})d^3x = \int_V \eta_{ij}{}^*(\mathbf{x})\eta_{ij}(\mathbf{x})d^3x = 1, \qquad (26a)$$

$$\partial_0\int_V \rho^{(n)}_{i,j}d^3x = -\oint_S \frac{1}{c}\mathbf{j}^{(n)}_{i,j}.d\mathbf{s}. \qquad (26b)$$

Schrödinger has used an infinite integral in Eq. (26a) and the quantization of the energy followed for quantum mechanics. In the UF Theory we are dealing with open systems in a finite volume, and the eigenvalues E_k are not a priori energy eigenvalues.

In the static case, the type of variation problems with the conditions Eq. (26a) and

$$\oint_S \frac{1}{c}\mathbf{j}^{(n)}_{i,j}.d\mathbf{s} = 0$$

which lead to a differential equation (22) has the general form

$$I(\eta^*,\eta) = \int_V d^3x\{h''(\nabla\eta^*(\mathbf{x})\nabla\eta(\mathbf{x}) + V(r)\eta^*(\mathbf{x})\eta(\mathbf{x})$$
$$+h'\mathbf{f}(\mathbf{A}(\mathbf{x}))(\eta(\mathbf{x})\nabla^*\eta(\mathbf{x}) - \eta^*(\mathbf{x})\nabla\eta(\mathbf{x}))\},\ r^2 = \mathbf{x}^2. \quad (27)$$

with a given scalar function V(r) and a vector function $\mathbf{f}(\mathbf{A}(\mathbf{x}))$ and with two constants h' and h''. However, the two constants are connected to each other,

$h'' = -h'^2$,

if a natural boundary condition on the surface S of the enclosed volume V exists, or alternatively, if the variation problem does not depend on the surface S. Such variation problems were investigated by the author in Refs. **[4].** The numerical technique was taken over by V. Marigliano Ramaglia and G. P. Zucchelli in Ref. **[8]** with considerable success.

Now, we turn to the physics of the radiation process. We suppose that a steady ground state of an H atom exists without radiation before the excitation. The assumption of a steady ground state corresponds to physical experiences and is not a "strong condition" on the physical system because an H atom never appears completely isolated from the rest of the world. The steady ground state corresponds to the lowest eigenvalues E_1 of Eq. (24) which is simultaneously the ionization energy

$E^{ionization} = -E_1$.

The numeric value of Planck's constant h is given by the known relation

$$h = q^2/2c \times (m'c^2/2E^{ionization})^{1/2} = q^2/2c\alpha_f, \quad (28)$$

α_f = fine structure constant, with

$E^{ionisation} = 13.59$ eV ,

and with

$m' = m_e m_P/(m_e + m_P)$,

of the H atom. In Eq. (28) an expression is used which allows a simple generalization to the relativistic case.

After the excitation $t>t_0$, the H atom system goes over to an excited state. The excited state is expressed as a superposition of solution of Eq. (24) with different eigenvalues

$$\eta_{i,j}(\mathbf{x},t) = \sum_k a_k \eta_{i,j;k}(\mathbf{x},t) = \sum_k a_k \eta_{i,j;k}(\mathbf{x}) \exp\{-iE_k(t-t_0)/\hbar\}. \quad (29)$$

The time dependent electric charge density in finite volume V is than given by

$$\rho^{(e)}_{i,j}(\mathbf{x},t) = q_i \eta_{i,j}^*(\mathbf{x},t)\eta_{i,j}(\mathbf{x},t) =$$
$$q_i \sum_{k,l} a_l^* a_k \eta_{i,j;l}^*(\mathbf{x})\eta_{i,j;k}(\mathbf{x})\exp\{-i(E_k - E_l)(t-t_0)/\hbar\}. \quad (30)$$

The electric charge density $\rho^{(e)}_{i,j}(\mathbf{x},t)$ oscillates with the frequencies

$\upsilon_{kl} = (E_k - E_l)/h$

and radiates electromagnetic field with these υ_{kl} in a finite space-time domain Ω. The observed decay time of exited states of atom is $\tau \sim 10^{-8}$ s. Therefore, the appearance of spectral lines with wave lengths

$\lambda_{kl} = c\upsilon_{kl}$

can only be observed in a moving finite space-time domain within a spherical sector $\Delta r^{e} \sim c\tau$ = 3m after the excitation t-t_0 > τ .

Fig. 1 shows that in accordance with the Eikonal theory the existence of photons is prohibited in all microscopic processes. In the New Model the emission process is connected to the source quantization of the UF without photons. Therefore, the quantization of the electromagnetic field with photons is not necessary and the usage of the classical field theory for electromagnetism is also allowed in atomic and nuclear physics. Furthermore, in the stationary Eq. (24) the contribution of the magnetic field $\mathbf{A}^{(e)}$ to the eigenvalues E_i is omitted. If this contribution is also considered in the eigenvalues problem, a splitting of spectral lines will appear. Conclusively, the observed multiple structure of spectral lines arises as a consequence of the presence of the magnetic field and not because the elementary particles have an intrinsic angular momentum (spin 1/2). The observed electromagnetic radiation in microscopic processes is connected with the source quantization of the UF with point-like and structure-less source particles having two kinds of Maxwell charges, see Refs. **[3, 5]**.

In case of the electron-neutrino ν_e = (e,p) we have the reduced mass

$m' = m_e/2$,

and the ionisation energy is

$E^{ionization} = 2m_e c^2$.

The Eq. (28) defines a second basic constant

$$h^0 = q^2/2c \text{ x } (m'c^2/2E^{ionization})^{1/2} = q^2/4c\sqrt{2}\,, \qquad (31)$$

and

h = 387.7xh^0.

The same constant h^0 arises for the proton-neutrino ν_P = (P,E) because the mass m_p is removing in Eq. (31) in course of the reduced mass

$m' = m_p/2$

and the ionisation energy

$E^{ionization} = 2m_p c^2$.

The constant h^0 determines also the size of the neutrinos,

$r = h^{0\,2}/(\pi e^2 m')$.

Since ν_e is responsible for the nuclear forces, h^0 determines with

r_{ν_e} = 7.03x10^{-14} cm ,

also the size of the nuclei. On the other hand is

r_{ν_P} = 3.83x10^{-17} cm.

The general variation of

$$I(A^{(e)},A^{(g)},\eta_i^*,\eta_i)= \int_\Omega (dx^4)\{L^T - L^0 - \sum_{i=1.4} (q_i A^{(e)}{}_\alpha - g_i A^{(g)}{}_\alpha) j_i^{(n)\,\alpha}(\eta_i^*,\eta_i)\}, \qquad (32)$$

where $L^T(x^\mu)$ and $j_i^{(n)\,\alpha}(x^\mu)$ is suitable expressed with the functions η_i and η_i*, I = 1, 4, and with a isoperimetric subsidiary conditions in a finite domain Ω

$G(\eta_i^*,\eta_i)$ =

$$\int_\Omega (dx^4) G(\eta_i^*(x^\mu), \eta_{i,\nu}^*(x^\mu)\eta_i(x^\mu), \eta_{i,\nu}(x^\mu), x^\mu) = \text{const}, \tag{33}$$

define Lagrange multipliers λ, see e.g. in book of M. Giaquinta, S. Hildebrandt, Ref. **[7],** Chapter 1. For the integrand in Eq. (33) we have to set

$$G(\eta_i^*(x^\mu), \eta_{i,\nu}^*(x^\mu)\eta_i(x^\mu), \eta_{i,\nu}(x^\mu), x^\mu) = \partial_\alpha j_i^{(n)\,\alpha}, \tag{34a}$$

and Eq. (33) is equivalent to

$$\int_V \rho_i^{(n)}(x,t)d^3x - \int_V \rho_i^{(n)}(x,t_0)d^3x + \int_\Omega (dx^4)\nabla.\mathbf{j}_i^{(n)} = 0 \text{ for i = 1, 4.} \tag{34b}$$

The Hamilton principle reads then

$$\delta \mathrm{I} = \delta \int_\Omega (dx^4)\{L\} = \delta|_{A^{(e)}} \int_\Omega (dx^4)\{L\} + \delta|_{A^{(g)}} \int_\Omega (dx^4)\{L\}$$
$$+ \sum_{i=1.4} \delta|_{\eta_i^*} \int_\Omega (dx^4)\{L\} + \sum_{i=1.4} \delta|_{\eta_i} \int_\Omega (dx^4)\{L\}, \tag{35}$$

where dx^4 is the invariant volume element, $A^{(e)\,\alpha}$ are the field quantities describing the electromagnetic field, $A^{(g)\,\alpha}$ the gravitational field and η_i, η_i^* the four kinds of elementary particles. The integration is to be performed in a finite space-time domain Ω. The quantum condition within the New Model, the source quantization, leads to isoperimetric subsidiary conditions generally written as the Eq. (33). The isoperimetric problem with natural boundary conditions, Eq. (34), is mathematically well defined and leads to Lagrange multipliers λ within the calculus of variation and the Lagrange formalism. The Lagrange multipliers λ, Ref. **[7]**,

$$\delta \mathrm{I}(A, \eta_i^*, \eta_i; \varphi_q) + \lambda \sum_{i=1.4} (\delta|_{\eta_i^*} + \delta|_{\eta_i}) G(\eta_i^*, \eta_i; \varphi_q) = 0, \text{ for all and } \varphi_q, \tag{36}$$

are eigenvalues of an open physical system in a finite domain Ω. In this context we connect the two constants h^0 and Planck's constant h with Lagrange multipliers. But it has to be mentioned that $\mathbf{j}_i^{(n)}$ may be expressed with the same functions η_i, η_i^* as $\rho_i^{(n)}$ if and only if Eq. (34b) is valid for definite numbers of particles N_i in a finite domain V at $t=t_0$ and no particles penetrate through the surface of V during a finite time interval.

Variation problems, defined by Eqs. (32), (34) and (36), build only a subclass of a great manifold of variation principles which occur in nature. A second class of variation principles arises if instead of a natural boundary conditions, Eq. (34b), a steady current of some types of particles is flowing across the surface $\partial\Omega$, containing within Ω a definite number of the other kinds of elementary particles. In this case Eqs. (34b) remain valid only for the latter particles. For instance, if the number of protons is only holding constant within Ω and a flow of ν_e = (e,p) is allowed, the problems of nuclei arise.

If the conditions of Eq. (34b) are valid for all four kinds of particles within Ω, the steady currents flowing through $\partial\Omega$ can only be the electromagnetic and the gravitational field. In case of laser, an electromagnetic wave with a definite frequency has to be considered flowing through the surface $\partial\Omega$ and within Ω there are the three types of particles e, p, P in a bound state.

Another specific problem is given at the equilibrium of bound systems of the three types of particles (e, p, P) with the electromagnetic field residing outside the surface at a temperature T. This equilibrium problem at given temperature

contains also statistics and corresponds to the black body radiation, where the constant h was found by Max Planck in 1900.
The source quantization of the UF with Eqs. (33)-(36) has also far reaching consequences for the context of interactions, because the creation and annihilation of the four EP with invariant properties is prohibited, as investigated by Szász in Refs. **[3, 5]**. The numbers of e, p, P and E are conserved in each frame and the electric charge and the gravitational mass do not change. The energy-mass-equivalence relation

$\Delta E = \Delta m^{i} c^{2}$,

is only valid for the inertial mass. The four EP are stable particles as observed in nature. A creation and annihilation of these four particles cannot arise in the context of the Unified Field Theory.
For instance, the spontaneous beta decay of a nuclei, the emission of an electron or a positron with an additional electron-neutrino ν_e = (e,p), has to be understood as an unstable state of nuclei within variation principles of the New Model. The beta decay corresponds to the so called weak interaction within the accepted Standard Model.
The observed unstable particles appear as temporary condensation of the neutrinos ν_e, ν_p and ν_{ep} on elementary particle systems. The temporary condensation with a participation of more than one proton-neutrino ν_p produces the so called strong interaction of particles, Ref**. [3]**. Consequently, there is no need for the introduction of any additional fundamental microscopic interactions within the New Model.
The four stable particles are the sources = quanta of UF, according to 2^{nd} and 3^{rd} basic hypotheses, and the Unified Field is the only interacting field, other particles than e, p, P, and E do not exist in nature. Therefore there is no need for an introduction of any other fundamental particles than the four elementary particles. Thus, there is no need for the introduction of quarks, of pre-quarks and of strings or anything else as fundamental particles for explanation of physics for $r > 10^{-18}$ cm.
A milestone of the UF Theory is the experimental verification of

$\Delta E = (m^{g} - m^{i}) c^{2} > 0$,

in composition dependent free fall by the author in Ref. **[6].** The value distribution of elementary charges according to the 7^{th} basic assumption and the equivalence of the velocity of light c and of gravity $c^{g} = c$ is axiomatically assumed. A next step of further simplification in basic physics could be an understanding of the UF without the separation

$A^{\alpha} = A^{(e)\ \alpha} + A^{(g)\ \alpha}$

with two different kinds of charges, then this is in a certain sense artificial. But at the moment, we are busy with the development of predictions of the New Model in comparison with them of the accepted Standard Model. At first, the Lagrange formalism of open systems in a finite space-time domain Ω has to be investigated in order to understand the connection of the basic constant h^{0} and Planck's constant h to different Lagrange multipliers λ.

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